METHOD FOR FINDING MIN AND MAX VALUES OF ERROR RANGE FOR CALCULATION OF MOMENT OF INERTIA

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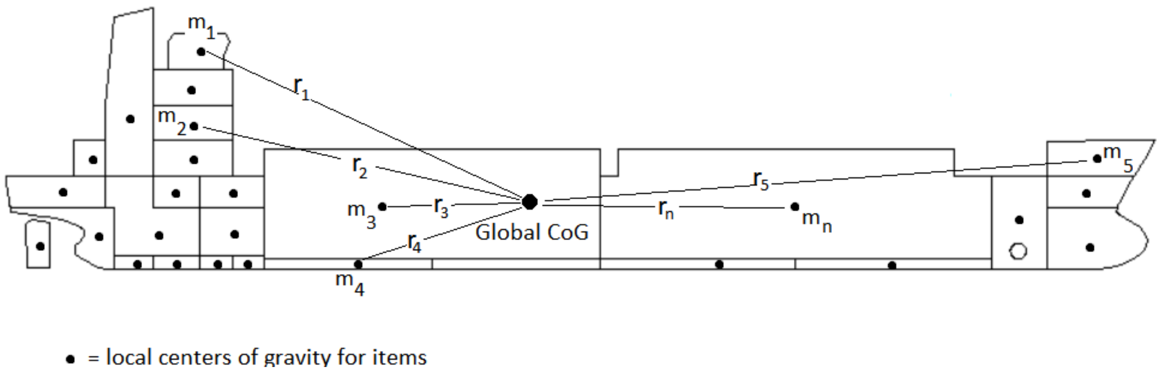
Moment of Inertia (MOI)

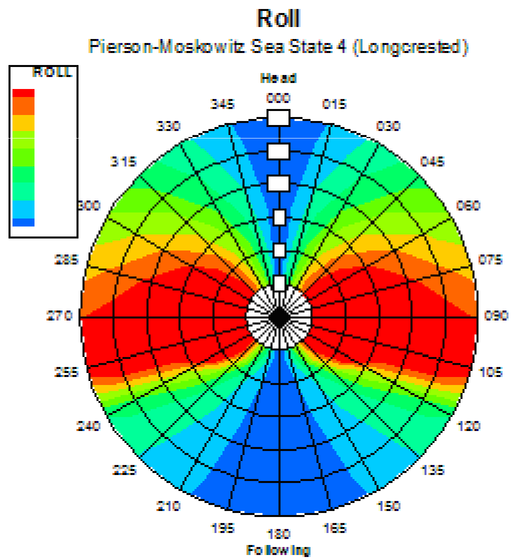
- This presentation will focus on
  - Impact of MOI on a vessels permformance
  - An alternative method to calculate MOI for vessels
  - A way to quantify the uncertainy range for the MOI calculation

- The case
  - A vessel composed of several items where weight, center of gravity and extension is known



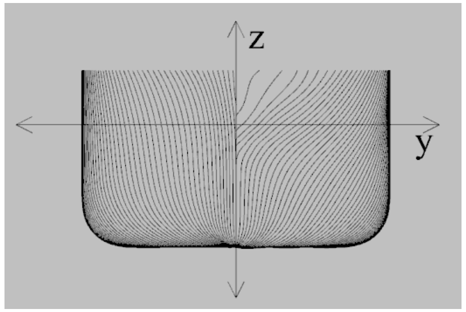
Impact of MOI on the Hydrodynamic Performance of a Vessel

- Added Resistance in Waves
- Model tests show trend of increase in resistance with increase in pitch inertia
- Racing sailors go to great lengths to reduce pitch inertia
- Seakeeping
- Roll (angle, resonant peak)
- Pitch
- Accelerations
- Motion Induced Sickness
- Operability



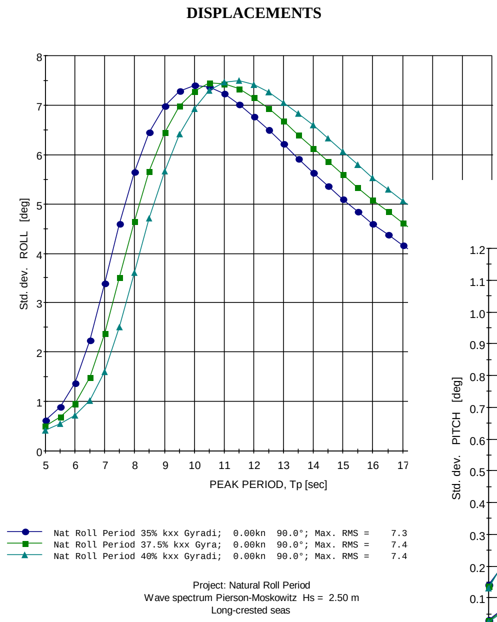
$$OI = \sum_{V_S}\sum_{\psi} P(V_S)\times P(\psi)\times O(V_S,\psi)$$

How Sensitive are the Motions to Variations in Gyradius?

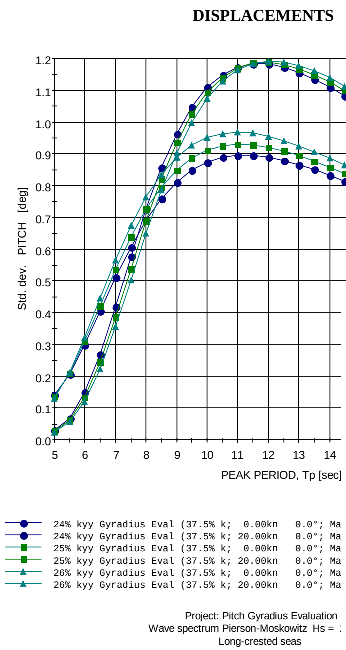


- Series 60 hull (LOA 121.25 m)
- Upper end of sea state 4 ($H_{1/3}$ = 2.5m, Bretschneider spectrum)
- Modal period between 5 and 20 seconds
- Speeds of 0 and 20 knots
- Roll gyradii of 35.0%, 37.5%, and 40.0% of BWL
- Pitch gyradii of 24%, 25%, and 26% of LWL

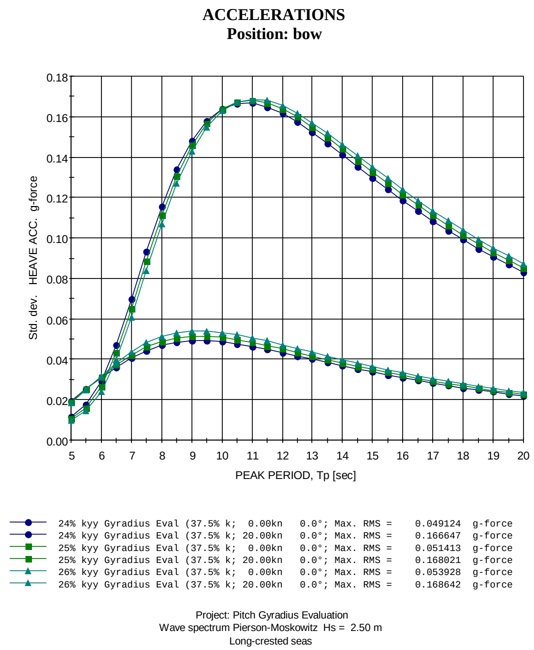
Seakeeping Results



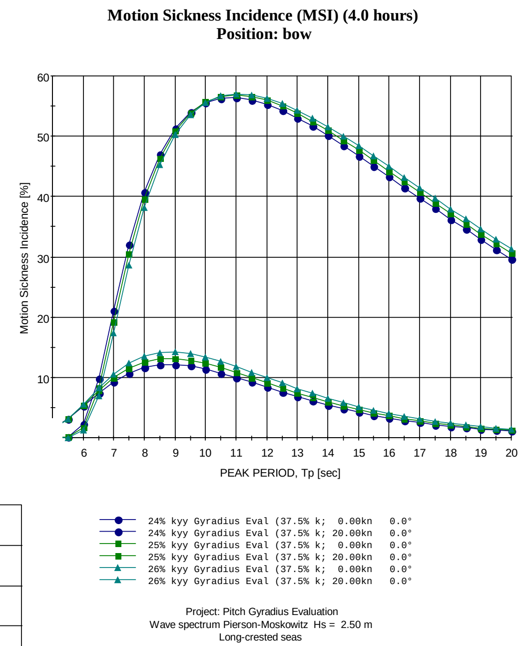
RMS Roll vs. Peak Period



Pitch vs. Peak Period



RMS Heave Acceleration @ Bow vs. Peak Period



MSI vs. Peak Period

General MOI Calculation Methods

- Accurate calculation throught 3D CAD
  - Not practical for early design
  - Labour intensive

- Combine parallel axis theorem with simplifying the shape of individial items to calculate MOI
  - Inaccurate
  - Error unknown
  - Many assumptions
    - Which simplified object should be selected to represent the actual object?
    - How do I account for density variations in the object?
    - Do I have the information needed to calculate the simplified object?
    - For a composed object (vessel): For which and how many items do I need to consider the self inertia (I_o)?

Proposing a New Approach

- Combine parallel axis theorem with a min/max approach for selfinertia by:
  - Find the minimum and maximum possible MOI for each individual item
  - Use the average value ***(max-min)/2*** as the MOI approximation for the individual items
  - Max error would then also be ***(max-min)/2***
  - Sort on max error values to find the items that needs further improvement to the approximation

Parallel Axis Theorem (Steiner's Theorem)

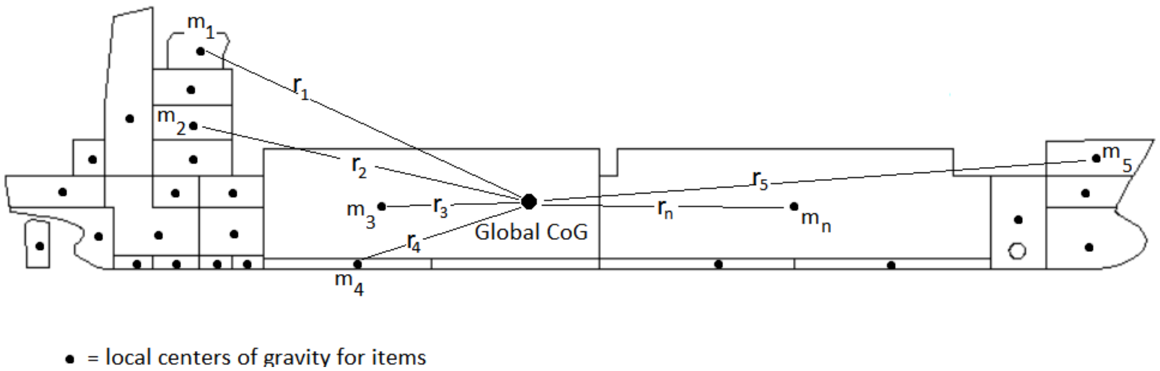
$$I_{total} = I_t + I_o \qquad\qquad I_t = \sum_{i=1}^{n} m_i r_i^2$$

Where I_t = transference inertia

I_o = self inertia

r = distance from global center of gravity to local center of gravity

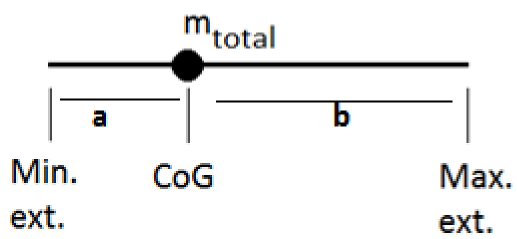
m = mass of item



I_t can be found from the parallel axis theorem, but I_o must be

Minimum Value of Self Inertia (I_o)

- The minimum value the self inertia I_o can obtain is 0.
- This follows from the mathematical definition of MOI and from the extreme situation of concentrating all mass of the object in a single point located at the local center of gravity for the object

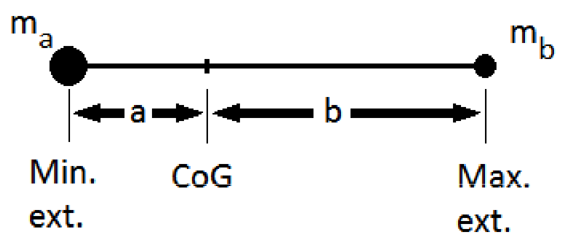


$$I_o = \int (a^2 + b^2)\, dm$$

a and ***b*** are 0 when ***dm*** > 0; and ***dm*** = 0 when ***a*** and ***b*** are > 0, thus I_o=0

Maximum Value of the Self Inertia (I_o)

- The maximum value the self inertia I_o can possibly have would be concentrating the mass at the end points of the extension of the item.



$$I_o = \int (a^2 + b^2) dm \quad \text{gives} \quad I_{o_{max}} = m_a a^2 + m_b b^2$$

Knowing that $m_{total} = m_a + m_b$ and that $m_a a = m_b b$

- thus we have enough information to solve and find the maximum I_o from the equations above

Approximating MOI and Finding the Accurate Error Range

- Once the minimum and maximum values of an object are found, the approximation of the self inertia would be the average value of the maximum and minimum value:

$$I_{o_{average}} = I_{o_{max\ error}} = \frac{(I_{o_{max}} - I_{o_{min}})}{2} = \frac{I_{o_{max}}}{2}$$

And thus the error range would be given by:

$$I_{o_{error\ range}} = I_{o_{average}} \pm \frac{I_{o_{max}}}{2}$$

The approximation of the MOI for the complete vessel would be found simply by adding the MOI found for each item and the total error range for the complete vessel would be found in similar way by adding the error ranges.

$$Total\ I_{o_{maxerror}} = \sum \left(\frac{I_{o_{max}}}{2}\right)$$

Example Calculation

Let's consider some sample data. The below is a snippet of data from a weight database of a vessel (VCG, LCG, TCG are referenced from the origin, not the global center of gravity):

Weight [kg]	VCG [m]	LCG [m]	TCG [m]	VCGmin [m]	VCGmax [m]	LCGmin [m]	LCGmax [m]	TCGmin [m]	TCGmax [m]
293	1.73	102.90	-0.95	0.38	2.14	101.06	107.02	-1.10	-0.69
1144	5.64	10.60	-0.09	4.70	5.79	9.83	10.91	-0.18	0.58
2994	5.37	60.33	0.47	4.82	7.14	55.64	62.70	-0.13	1.04
369	7.90	89.55	0.12	7.66	8.73	84.88	94.29	-0.49	0.73
96	0.00	0.00	0.00	-2.08	0.21	-2.86	0.58	-0.27	0.10
752	8.50	1.50	-0.60	7.92	10.58	-1.93	5.00	-1.57	-0.25
3523	5.02	41.27	0.40	3.42	6.85	36.41	44.12	-0.49	0.99
8579	9.69	40.00	0.00	7.87	10.84	35.57	44.25	-0.37	0.66
68	10.30	33.00	0.00	9.56	12.14	32.70	35.69	-0.34	0.29
211	10.16	3.88	0.00	9.36	10.61	0.63	7.59	-0.72	0.91
121	15.93	92.82	-6.62	15.36	16.22	89.05	95.58	-7.36	-5.62
2325	4.85	61.30	0.63	4.40	6.89	59.02	62.21	-0.21	0.71
887	7.10	90.60	0.15	6.08	8.94	86.19	92.41	-0.02	0.71
294	9.83	26.70	-9.73	8.98	9.99	22.25	26.94	-10.42	-8.90

Applying the Formulas to the Sample

Roll calculation sample:

m_{total}	=	Total item weight
r	=	Distance between global CoG and item (local) CoG
a	=	Distance from item (local) CoG to item's start point
b	=	Distance from item (local) CoG to item's end point
m_a	=	Part weight transposed to item's start point
m_b	=	Part weight transposed to item's end point
I_t	=	Transference inertia (m_{total} * r^2)
I_o	=	Self inertia
I_{total}	=	Total inertia (I_t + I_o)
I_{error}	=	Max total inertia error according to method

m_{total}	r	a	b	m_a	m_b	I_t	I_o	$I_{x - total}$	$I_{x - error}$
293	8.0	1.80	1.43	129.8	163.4	18592	377	18969	2.0 %
1144	4.0	0.88	1.19	656.6	487.2	18277	601	18879	3.2 %
2994	4.3	1.91	1.44	1287.2	1707.1	54866	4116	58983	7.0 %
369	1.7	1.23	1.67	212.8	156.3	1107	377	1484	25.4 %
96	9.6	1.70	1.07	37.2	58.9	8910	87	8998	1.0 %
752	1.3	1.66	0.88	260.0	492.0	1278	550	1828	30.1 %
3523	4.6	1.75	0.07	143.4	3379.4	75331	229	75560	0.3 %
8579	0.1	0.70	1.80	6181.7	2397.4	47	5382	5429	99.1 %
68	0.7	1.22	1.50	37.2	30.3	30	62	92	67.0 %
211	0.5	0.73	1.39	138.2	72.3	59	107	165	64.6 %
121	9.2	0.85	0.92	62.7	58.0	10160	47	10207	0.5 %
2325	4.8	1.01	1.71	1460.2	864.3	53881	2015	55895	3.6 %
887	2.5	0.33	0.76	616.2	270.7	5686	113	5798	1.9 %
294	9.8	1.44	1.87	166.2	128.1	28132	396	28528	1.4 %
21655						**276357**	**14459**	**290816**	**5.0%**

Finding and Improving the Most Inaccurate Items

- The most inaccurate items are found simply by sorting your items with regard to the self inertia max error range calculated.

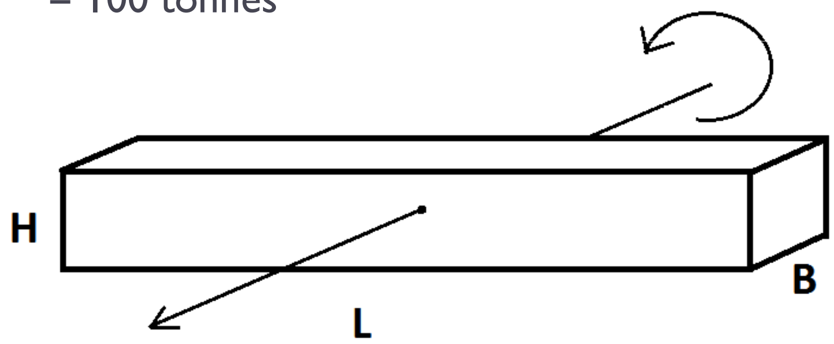
- Once these items are identified, you can improve the accuracy for these items either by

  - Calculating the self inertia accurately by making an exact model in CAD and calculate MOI from integration on the geometry; or

  - Dividing the item into several sub-items, thus reducing the total inaccuracy

The Power of Dividing an Item into Sub Items (1)

- Consider a "vessel" built as a solid, homogenous box with the following dimensions (metric units applied, but quantities are irrelevant here):
  - L (length) = 80 m
  - B (beam) = 10 m
  - H (height) = 10 m
  - M (mass) = 100 tonnes



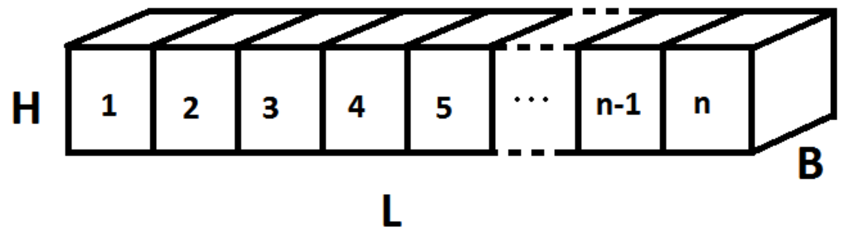
$$I_{yy} = k(L^2 + H^2) \qquad k = \frac{1}{12}$$

Given the parameters above, I_{yy} = 54167 tonnes-m^2.

The Power of Dividing an Item into Sub Items (2)

- It may be shown that when dividing this box into sub boxes and using the parallel axis theorem, combined with the self inertia of the sub boxes, the part of transference inertia (I_t) and the part of self inertia from the sub boxes (I_o) may be found from the following formulas:

$$I_o = kM\left[\left(\frac{L}{n}\right)^2 + H^2\right] \qquad I_t = 2M\sum_{i=1}^{n/2}\left(\frac{Li}{n} - \frac{L}{2n}\right)^2$$



Number of sub boxes	MOI from transference	Self Inertia	Error from neglecting self inertia
1	0	54167	100.0 %
2	40000	14167	26.2 %
4	50000	4167	7.7 %
8	52500	1667	3.1 %
16	53125	1042	1.9 %

Advantages and Disadvantages of the Method

- Advantages
  - The advantages of this method may be summarized as follows:
  - The method will provide an exact error range; you will know the minimum and maximum MOI the object can possibly have.
  - You can find the items that contribute most to the overall error range and target these items to improve your estimate, making sure you work no more and no less than needed to fulfill the accuracy needed.
  - You do not need to define and position simplified geometric objects to deal with the self inertia of individual items and the uncertainty that these simplifications imply.

- Disadvantages
  - The biggest disadvantage of the method is that it requires the extensions in all three directions for the items involved in the calculation. In case of extracting information from a CAD system this might be as easy as defining the output to include these quantities, but in case of manual input, this might represent a significant amount of work.